# Assignment 7

1. Apply two steps of Newton's method to find a simultaneous root of the following system of two algebraic equations starting with the approximation $x = y = 0.5$.

$$x^2 + 4y^2 - x + y - 1 = 0$$
$$3x^2 + 2y^2 + \mathrm{x} - y - 2 = 0$$

2. Apply one step of Newton's method to find a simultaneous root of the following system of three algebraic equations starting with the approximation $x = y = -0.5$ and $z = 1$.

$$3x^2 + x - xy - 2y - 1 = 0$$
$$2x + 2y^2 + xy - y + z - yz - 2 = 0$$
$$y - 2\mathrm{z} + yz + 3z^2 = 0$$

3. Apply one step of Newton's method to find a simultaneous root of the following system of two algebraic equations starting with the approximation $x = 1$ and $y = 1.5$ .

$$\sin(x) + 2\cos(xy) = 1$$
$$\sin(xy) - 2\cos(y) = 1$$

4. Recall the difference between Newton's method and the secant method for a single algebraic equation in a single variable. Suppose instead, you did not have the derivative. How would you generalize the secant method for two algebraic equations in two variables, or $n$ algebraic equations in $n$ variables? Recall that a plane is defined by three points.

5. Suppose you have the ordinary differential equation $y^{(1)}(t) = \sin(y(t))$ and you know that $y(0) = 1$ and $y(0.1) = 1.086355758991046$. Use a cubic spline to approximate $y(0.05)$.

6. Suppse you knew that $y(a) = y_a$, $y(b) = y_b$, $y^{(1)}(a) = y_a^{(1)}$, $y^{(1)}(b) = y_b^{(1)}$, $y^{(2)}(a) = y_a^{(2)}$, $y^{(2)}(b) = y_b^{(2)}$. Write down the system of linear equations that would find the quintic (degree five) polynomial that satisfies these conditions.

7. Using Euler's method, approximate $y(1)$ with $h = 0.2$ and again with $h = 0.1$ for the initial-value problem defined by

$$y^{(1)}(t) = 2y(t) + t - 1$$
$$y(0) = 1$$

8. In Question 7, you approximated $y(0.2)$ with $h = 0.2$, and $y(0.1)$ with $h = 0.1$. The correct solutions to sixteen significant digits are $y(0.2) = 1.268868523230952$ and $y(0.1) = 1.116052068620128$. Show that the error of one step of Euler's method is $O(h^2)$ by showing that the error of your approximation at $t = 0.1$ is approximately one quarter the error at $t = 0.2$.